1. Objectives

Wavefront measurement is an essential step toward ensuring the delivery of high-quality images in astronomical instruments. There are several types of wavefront sensor. Some measure the phase directly, e.g. through interferometric effects, while others measure gradients of the phase, like Shack-Hartmann or curvature sensors. In this lab we will focus on the Shack-Hartmann sensor, which is commonly used in adaptive optical systems in astronomy.

This laboratory exercise has several objectives:

- Understand the operating principle of a Shack-Hartmann wavefront sensor.
- Understand the purpose and design of a relay system for reimaging the lenslet spots onto the detector.
- Assemble and align a working Shack-Hartmann sensor.
- Measure spot displacements and translate those displacements into wavefront slopes.

This laboratory activity mixes written work in this worksheet with hardware assembly and experimentation. You will be working in small groups; be sure to engage your entire group in discussion, and be respectful of others’ learning. Ask for assistance from the facilitators — but don’t expect to be given answers!

Do not touch the optical surfaces directly. Never look directly into the optical fiber tips!

While you are free to adjust the placement of the components used in the lab, please do not adjust the post collars without consulting a facilitator first.

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2. Sensing Phase Gradients: Tip and Tilt

Lenses act to alter the wavefronts of electromagnetic waves. In the ray optics approximation, lenses can act to bring ray bundles to a focus. For example, rays emanating from a point in the object plane will converge to a point in the image plane. Rays that are parallel upon entering a lens will converge at a point a distance $f$ from a thin lens, its focal length. In the wave optics view, initially flat wavefronts are converted to spherical waves which converge at a point in the focal plane (Fig. 1). Remember that rays are perpendicular to phase fronts in the geometric optics approximation.

![Fig. 1.— A positive thin lens brings initially parallel rays into convergence at the focus (dotted). It acts to convert flat wavefronts into spherically converging ones (solid).](image1)

If a wavefront is tilted relative to the optical axis, after the lens the light will converge at a point that is displaced in the focal plane (Fig. 2). This can be used as a simple wavefront sensor to detect tipped or tilted wavefronts.

![Fig. 2.— Tilted wavefronts converge at displaced points in the focal plane.](image2)
Q1: By what distance \( d \) is the focus displaced for a lens with focal length \( f \) given a wavefront tilt angle \( \theta \)? (See Figure 2.) In the small angle approximation?

A:

More complicated wavefronts can be decomposed into a superposition of modes. One dimensional functions might be described by a Fourier series of sines and cosines. In optics, it is more common to use a two-dimensional basis of modes, for example the so-called Zernike modes. The average incidence angle of the wavefront is described by the “tip” and “tilt” Zernike modes, and they result in a displacement of the focused wavefront in the focal plane of the lens. See Appendix 5. If we monitor the displacement of the image in the focal plane, we can sense the tip and tilt modes. A lens is a tip/tilt wavefront sensor.

Fig. 3.— (left) The Point Grey Flea sensor. The light gray area at the center is the CMOS detector array. Each pixel is 6 \( \mu \)m in size. (right) The 25 mm camera lens. The lens has a C-mount which screws into the image body. This component has a built-in aperture stop, which can be adjusted by a ring on the body. The C designation means closed. There is also an adjustable focus barrel.

<table>
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<th>component</th>
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<th>symbol</th>
<th>value</th>
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<tr>
<td>lenslet array</td>
<td>pitch</td>
<td>( p )</td>
<td>300 ( \mu )m</td>
</tr>
<tr>
<td>lenslet array</td>
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<td>( f_1 )</td>
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<td>( f_2 )</td>
<td>75 mm</td>
</tr>
<tr>
<td>lens</td>
<td>focal length</td>
<td>( f_3 )</td>
<td>25 mm</td>
</tr>
<tr>
<td>detector</td>
<td>pixel scale</td>
<td>( \Delta x )</td>
<td>6.0 ( \mu )m</td>
</tr>
</tbody>
</table>

Table 1: Summary of size scales in the optical system.
**Experiment 1:** You have a 25 mm lens in a C-mount attached to a CMOS detector, and a collimated light source. The goal of this experiment is to verify the expected displacement of the spot image on the detector given a measured change in the angle of the input phase front.

1. Set a post holder base against the rear of the collimated source base plate. This will act as a guide against which you can move the light source.

2. Adjust the iris aperture internal to the 25 mm lens to the fully open position, f/1.4.

3. Form an image of the source on the detector in the focal plane of the lens.

4. Note that translation of the source perpendicular to the detector does not displace the focused image spot.

5. Record the position of the image on the detector.

6. Change the angle of the incoming wavefront by a measured amount. You may find a protractor will be helpful here.

7. Measure the displacement of the image on the detector, and compare it to your expectation.
3. Shack-Hartmann Basic Principles

Ideally we would like to sense the phase of the incoming wavefront directly. This is often for the purpose of sensing aberrations, which astronomers may want to correct in an adaptive optics system, for example. It may be possible to instead recover the useful information in the phase front (i.e. its aberrations) using spatial derivatives of the phase front. A lens can sense the gradient, or slope, of the phase across its entrance aperture. More generally, an array of slope sensors can therefore recover first-order derivatives of the wavefront.

A Shack-Hartmann sensor is an array of individual slope sensors. It is often implemented using a microlens array. The focused spots from each lenslet are imaged onto a CMOS detector. The displacement of the spots on the CMOS detector can then be easily measured (Figs. 4 and 5). The microlens array senses the average slope of the phase across each lenslet. It can therefore sense phase gradient information down to scales comparable to the size of the lenslets, known as the lenslet pitch.

![Fig. 4.— A lenslet-based Shack-Hartmann device translates local phase gradients to spot displacements. For a flat incoming wavefront as shown in the left panel, the lenslets focus the light to a regular array of spots. The spot locations are shown in two dimensions on the right panel. The positions of the spots given a flat incoming wavefront are the “reference positions.”](image)

While the Shack-Hartmann sensor is sensitive to phase gradients, it is possible to use this information to identify low-order modes of the wavefront phase (excepting piston). For example, the tip and tilt modes discussed earlier have well-defined spot displacement patterns (Fig. 6).
Fig. 5.— A lenslet-based Shack-Hartmann device senses wavefront slopes across each subaperture via the displacement of spots focused on a detector. The wiggly line on the left represents an aberrated phase front. This is approximated by the average gradient in small patches, which is overlaid in the middle wavefront, and shown alone just before the entrance to the microlens array. These local tilts result in displacements in the detector plane. On the figure at right, open circles represent the reference positions of the spots in the absence of aberrations.

Fig. 6.— Displacement of spots for tip and tilt modes. The spot displacements are all the same, since the phase gradients across each subaperture are equivalent.
**Q2:** What would the displacement pattern look like if a positive lens were placed in the collimated beam entering the sensor?

**A:** *(The open circles represent the reference spot positions for a flat incoming wavefront. Overlay this figure with the expected dot pattern.)*

![Displacement pattern diagram](image)

You have a microlens array in a rectangular mount (Fig. 7). The array is a ThorLabs MLA300-14AR, which is a 10 mm x 10 mm array with a square grid of microlenses, having a 300 µm pitch. Each lenslet has a 14 mm focal length.

![Microlens array](image)

![75 mm focal length lens](image)

**Q3:** What would happen if the spots were focused directly on the detector? How many pixels between each spot, and how many spots will be imaged?

**A:**

You also have a CMOS detector, which has a 752 x 480 pixel array with 6.0 µm square pixels.

**Q3:** What would happen if the spots were focused directly on the detector? How many pixels between each spot, and how many spots will be imaged?

**A:**
Experiment 2: Determining whether spots can be focused directly on the detector.

1. First, place the lens cap on the 25 mm C-mounted lens attached to the detector.
2. Remove the 25 mm lens from the system and place it carefully aside.
3. Do not remove the microlens array from its mount.
4. Calculate where the detector needs to be to obtain focused spots.
5. Attempt to obtain focused spots on the detector. Can they be obtained? Why or why not?

4. Shack-Hartmann Sensor Design

As you have seen, the CMOS detector pixel size is much smaller than the lenslet pitch. This means the spots will be far apart on the detector, which is an inefficient use of the detector. Moreover, it can be challenging to focus the spots on the detector given packaging constraints. We can solve both packing issues using an optical relay.

Our relay system uses a pair of lenses to reimage the microlens array’s focused spots onto the detector. This is done in such a way as to more densely pack the spots onto the CMOS detector. Figure 8 shows a schematic raytrace of the microlens array and the optical relay.

Follow the raytrace in the diagram. Note the chief and bounding rays for each lenslet (solid, dashed, and dotted lines). Between the lenslets and the first relay lens, these rays are parallel. Note that parallel rays come to a focus one focal length \(f_2\) after the first relay lens. These positions are also one focal length away \(f_3\) from the second relay lens, and therefore emerge as parallel rays upon exiting that lens. The chief and bounding rays again come to a focus on the detector, successfully reimaging the lenslet spots onto the CMOS detector.

Q4: What do you expect the physical spacing between spots to be on the detector, in microns? In pixels? Note values for the various length scales are given in Table. 1.
A:
Q5: You calculated the physical displacement of a spot in the lenslet’s focal plane for a beam deviation of angle $\theta$. What will this displacement be in the detector plane, in pixels?
A:

Q6: What will this displacement be as a fraction of the spacing between spots?
A:

**Experiment 3:** Assemble the Shack-Hartmann sensor laid out in Fig. 8 using the components at hand. Use a ruler to assist you in placing the optics at the appropriate distances.
Fig. 9.— Adjustable aperture stop inside mount (Edmund Optics NT53-914).

Q7: What is the purpose of the aperture stop? Verify your hypothesis by removing and replacing the stop from its post holder and examining the effects on the detected image.
A:

Q8: How big should the aperture stop be?
A:

Experiment 4: Obtain an image of the focused spots on the CMOS detector. Measure the distance between the spots in pixels, and compare to your calculation above. If they disagree, what might be the cause(s)?
5. Exploring the Sensor

Now that you have a working sensor, you can perform some quantitative measurements, sensing aberrations from a variety of sources.

**Experiment 5:** Can you observe displacements from fluctuations the optical path through a transparent material, like plastic?

**Experiment 6:** Locate some sources of fixed aberration, for example trial lenses from an optician’s kit. Use the sensor to both qualitatively and quantitatively examine spot displacements.

1. Save an unaberrated image of the spots as a TIFF file.
2. Place a cylindrical lens in the beam, and examine the spot displacements across the detector. Rotate the lens.
3. Place a trial lens in the beam, and try to measure the spot deviations.
4. Qualitatively compare the field dependence of the spot displacements with what you expect. For a positive lens, you may recall your answer to question 2.
5. Measure the field dependence of the spot deviation across the detector. Comparison between TIFF files may be useful.
6. Compare the displacements you measure with those that you quantitatively expect.

**Extra Challenge:** Ask your facilitator to place an unknown aberration into your system, and attempt to approximately reconstruct the wavefront using your wavefront sensor. Document your group’s solution and turn it in to your facilitator.
Fig. 10.— An example of an assembled Shack-Hartmann wavefront sensor.

A. Zernike Polynomials

The following notes are for your reference and are not required for the lab. Zernike polynomials are a radial basis set that is often used to describe wavefronts. Arbitrary wavefronts can be decomposed into Zernike modes. The Zernike polynomials are:

$$W(\rho, \theta) = \sum_{n} \sum_{m} C_{m}^{n} N_{n}^{m} Z_{n}^{m}, \quad (A1)$$

where $m$ and $n$ are nonnegative integers with $n \geq m$, $\theta$ is the azimuthal angle, and $\rho$ is the radial distance, with $0 \leq \rho \leq 1$. $C_{m}^{n}$ is the coefficient for each Zernike term, and is the root-mean-square for that aberration type. For $m \geq 0$:

$$Z_{n}^{m}(\rho, \theta) = R_{n}^{|m|}(\rho) \cos(m, \theta), \quad (A2)$$

for $m < 0$:

$$Z_{n}^{m}(\rho, \theta) = R_{n}^{|m|}(\rho) \sin(m, \theta) \quad (A3)$$

Zernike polynomials have the property of being limited to a range of -1 to +1, i.e. $|Z_{n}^{m}(\rho, \theta)| \leq 1$. The radial polynomials $R_{n}^{m}$ are defined as

$$R_{n}^{m}(\rho) = \sum_{k=0}^{(n-m)/2} \frac{(-1)^{k} (n-k)!}{k! ((n+m)/2-k)! ((n-m)/2-k)!} \rho^{n-2k}. \quad (A4)$$
when $n - m$ is even, and are 0 when $n - m$ is odd.

$$N_n^m = \sqrt{\frac{2(n + 1)}{1 + \delta_{m0}}}$$  \hspace{1cm} (A5)

$\delta_{m0} = 1$ when $m = 0$, and $\delta_{m0} = 0$ otherwise.

Figure 11 shows a graphical representation of the first few Zernike modes, with the top of the pyramid corresponding to the lowest order mode, which is just a constant (piston term). The next row are tip and tilt aberrations, followed by the row with astigmatism and defocus. Table 2 lists the first 15 Zernike terms.

Fig. 11.— Representation of Zenike modes. The colors correspond to the height of the deviation from a flat front wavefront. See Table 2 for the common names of these terms as well as their mathematical expression. Source: Wikimedia Commons
<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>N^n_m</th>
<th>Z^n_m</th>
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<td>0</td>
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<td>1</td>
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</tr>
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<td>1</td>
<td>-1</td>
<td>2</td>
<td>(\rho \sin \theta)</td>
<td>Tip</td>
</tr>
<tr>
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<td>2</td>
<td>(\rho \cos \theta)</td>
<td>Tilt</td>
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<td>(2\rho^2 - 1)</td>
<td>Defocus</td>
</tr>
<tr>
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<td>Astigmatism, axis (0^\circ &amp; 90^\circ)</td>
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<td>(\rho^3 \sin 3\theta)</td>
<td>Trefoil</td>
</tr>
<tr>
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<td>((3\rho^3 - 2\rho) \sin \theta)</td>
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</tr>
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<tr>
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Table 2: Zernike Aberration Terms