CRASH COURSE IN INTRO TO OPTICS

Dunlap Instrumentation Summer School
July 25, 2017

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Some of my highs and lows navigating a career in astrophysics

(totally not to scale- does not necessarily correlate with personal life)

S. Wright – July 24, 2017
Astronomical Instrumentation

• The detector and readout and “drive” electronics
• Opto-mechanical layout (telescopes, lenses, mirrors, filters, gratings, etc.)
• Enclosure and cooling (i.e., detector, instruments components)
• Signal processing hardware (e.g., amplifiers, ADU converters)
• Motion control and “housekeeping” systems (e.g., motorized mechanisms, temperature control, monitoring devices)
• Electronic interfaces
• Computers and peripherals
• Image/data display and data processing
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Lecture Outline

• **Light and its interaction with physical objects**
  – Ways of thinking about light
  – Diffraction
  – Reflection and Refraction
  – Scattering
  – Absorption

• **Basic Optics: raytracing, lenses, & telescopes**
  – Thin lenses
  – Images, focal planes, lensmaker formula
  – Telescope, f-numbers
  – Pupils
  – Aberrations
I. Light and its interaction with physical stuff
Light: A Carrier of Astrophysical Information

• Light exhibits the quantum mechanical property of wave-particle duality.

• We consider the wave nature, representing a probability distribution for photon location.

• We also consider the particle interaction of individual photons with matter.
Method/measurements used dictates ways we think of light

• **Photons** – particles with energy $h \nu$
  - Important for calculating detector efficiencies, considering scattering processes, beamsplitting, blackbody radiation

• **Waves** – with interference, vibrate charges in a substance have a polarization.
  - Important for diffraction limit, seeing, gratings, diffraction spikes, polarization effects in scattering

• **Rays** – representing the path of photons (or equivalently wavefront normals)
  - Important for thinking about optics, image quality, aberrations, etc. (not as fundamental as particle and waves, but useful analytically/geometrically)
Light as a wave

- Light is a transverse wave
- Maxwell’s equations is an EM wave that propagates in z direction with an E and B field
- Characterized by two complex numbers, e.g., $E_0^x$, $E_0^y$ or four real real ones ($I, Q, U, V$)—the Stokes parameters
- Intensity is proportional to $|E_0^x|^2 + |E_0^y|^2$

\[
\begin{align*}
\vec{E}_x(z, t) &= E_{0x} \cos(kz - \omega t) \hat{x} \\
\vec{E}_y(z, t) &= E_{0y} \cos(kz - \omega t + \epsilon) \hat{y}
\end{align*}
\]
Electromagnetic Radiation

- The quantization of light, $E=\hbar \nu$, means that each frequency or type of photon requires different materials and methods for detection.
### Energy of photons

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Wavelength (nm)</th>
<th>Energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma rays</td>
<td>&lt;1 pm</td>
<td>&gt;1.2 MeV</td>
</tr>
<tr>
<td>Soft gamma rays</td>
<td>0.001–0.01</td>
<td>120–1200 keV</td>
</tr>
<tr>
<td>X-rays</td>
<td>0.01–1</td>
<td>1.2–120 keV</td>
</tr>
<tr>
<td>Soft X-rays</td>
<td>1–10</td>
<td>120 eV–1.2 keV</td>
</tr>
<tr>
<td>EUV</td>
<td>10–100</td>
<td>12 eV–120 eV</td>
</tr>
<tr>
<td>UV</td>
<td>100–400</td>
<td>3–12 eV</td>
</tr>
<tr>
<td>Visible</td>
<td>400–700</td>
<td>1.7–3 eV</td>
</tr>
<tr>
<td>Near IR</td>
<td>1–3 μm</td>
<td>1.2–1.7 eV</td>
</tr>
<tr>
<td>Mid IR</td>
<td>3–10 μm</td>
<td>0.4–1.2 eV</td>
</tr>
<tr>
<td>Thermal IR</td>
<td>10–1000 μm</td>
<td>0.12–0.4 eV</td>
</tr>
<tr>
<td>Millimeter</td>
<td>1–3 mm</td>
<td>$1.2 \times 10^{-4}$–0.12 eV</td>
</tr>
<tr>
<td>Radio</td>
<td>&gt;1 cm</td>
<td>&lt; $1.2 \times 10^{-4}$ eV</td>
</tr>
</tbody>
</table>
Geometrical vs. Physical Optics

- **Geometrical**
  - Traces “rays” which do not interact with one another
  - Purely Geometric (e.g., Snell’s law, law of reflection)
  - Intersection of individual rays defines a focal plane and gives an image

- **Physical**
  - Wave nature of light
  - Wavefronts (rather than rays)
  - Wave propagation and interference determine illumination
  - Interference, diffraction, etc.
Fundamentals of light

- Light travels in a **straight line** in constant-refractive-index medium at speed $c/n$
- Refractive index $n$ is 1.0 in vacuum, and is related to the permittivity ($\varepsilon$) and permeability ($\mu$) of material: $c^{-1}=\sqrt{\varepsilon_0\mu_0}$; $v^{-1}=\sqrt{\varepsilon\mu}$; $n=\sqrt{\varepsilon\mu/\varepsilon_0\mu_0}$

<table>
<thead>
<tr>
<th>Material</th>
<th>Index (n)</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>1.000274</td>
<td>274ppm different from vacuum</td>
</tr>
<tr>
<td>water</td>
<td>1.333</td>
<td>Similar for eyeball</td>
</tr>
<tr>
<td>quartz</td>
<td>1.458</td>
<td>Aka, fused silica</td>
</tr>
<tr>
<td>BK7 glass</td>
<td>1.52</td>
<td>Common optics for lenses</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.419</td>
<td></td>
</tr>
<tr>
<td>Silicon</td>
<td>~3.5</td>
<td>CCD material, for instance</td>
</tr>
</tbody>
</table>
Reflection

- Reflection off a flat surface follows a simple rule:
  - angle in (incidence) equals angle out
  - angles measured from surface “normal” (perpendicular)
Reflection

• Also consistent with “principle of least time”
  
  – Fermat’s principle
  
  – If going from point A to point B, reflecting off a mirror, the path traveled is also the most expedient (shortest) route
Curved mirrors

- **What if the mirror isn’t flat?**
  - light still follows the same rules, with *local* surface normal

- **Parabolic mirrors have exact focus**
  - used in telescopes, backyard satellite dishes, etc.

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Refraction

- **Light bends at interface between refractive indices (n)**
  - bends more the larger the difference in refractive index
  - can be effectively viewed as a “least time” behavior - get from A to B faster if you spend less time in the slow medium

Snell’s Law:

\[ n_1 \sin \Theta_1 = n_2 \sin \Theta_2 \]
Internal reflection

• At critical angle, refraction no longer occurs
  – thereafter, you get total internal reflection
    \[ n_2 \sin \Theta_2 = n_1 \sin \Theta_1 \; ; \; \Theta_{\text{critical}} = \sin^{-1}(n_1/n_2) \]
  – for glass, the critical internal angle is 42°
  – for water, it’s 49°
  – a ray within the higher index medium cannot escape at shallower angles
  – Total internal reflection often exploited in fibers

\[
\begin{array}{c|c}
 n_1 & 1.0 \\
 n_2 & 1.5 \\
\end{array}
\]
Optical design we consider every surface

- Let’s consider a thick piece of glass ($n = 1.5$), and the light paths associated with it
  - reflection fraction = $[(n_1 - n_2)/(n_1 + n_2)]^2$
  - using $n_1 = 1.5$, $n_2 = 1.0$ (air), $R = (0.5/2.5)^2 = 0.04 = 4\%$

incoming ray (100\%)

8\% reflected in two reflections (front & back)

4\% 4\% 4\% 0.16\% 92\% transmitted
Polarization at interfaces

• Incident light waves when hitting a medium wiggle charge (electrons) - direction of wiggle dependent on wave’s electric field

• Linear polarization important for both reflection and refraction at medium

P-polarized (from the German “parallel”) light has an electric field polarized parallel to the plane of incidence, while s-polarized (from the German “senkrecht”) light is perpendicular to this plane
Example polarization at interfaces

- Metal reflective coatings do not substantially affect polarization
  - Metals have free electrons

- Polarizing filters on camera or sunglasses can knock out reflections off glass or water – allowing only particular polarizations to be transmitted
Scattering of light

• Light also scatters off junk – air particles, dust, dirty mirrors, rough optical surfaces
  – Instrumentalist need to pay attention to surface quality of optics and particles within the instrument to reduce scattered light

• Scattering is a strong function of wavelength, generally scales as Rayleigh scattering as $\lambda^{-4}$
  – E.g., at sea level, dry air back scatters non-isotropic (e.g., sky is darkest blue 90 degrees away from Sun)
Optical coatings

- Anti-reflection (AR) coatings work by interference patterns
  - Transmission/Reflectivity a function of polarization (P/S)
Absorption

- Not all light is reflected or transmitted, some is absorbed
- Aluminum is a typical reflective coating (~92% R), Silver is better 96% R. Both good at broad $\lambda$, but gold better at infrared
  - Al chosen for most telescopes cause it does not tarnish

Reflectivity (R), Transmission (T), and Absorption(A) for P and S polarizations

<table>
<thead>
<tr>
<th></th>
<th>R(P)</th>
<th>T(P)</th>
<th>A(P)</th>
<th>R(s)</th>
<th>T(s)</th>
<th>A(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>88.7 %</td>
<td>0%</td>
<td>11.3 %</td>
<td>94.3 %</td>
<td>0%</td>
<td>5.8 %</td>
</tr>
<tr>
<td>AR @ 532 nm</td>
<td>0.5 %</td>
<td>99.5 %</td>
<td>0 %</td>
<td>0.3 %</td>
<td>99.7 %</td>
<td>0 %</td>
</tr>
</tbody>
</table>
Physical Wave Optics

- **Interference**: waves interfere constructively (in phase) and destructively (out of phase)

- **Diffraction**: when a wavefront passes through a narrow aperture, it will spread out on the other side
Resolution of an imaging system

- Consider an off-axis point y for a single 1-d slit with width D and diffraction pattern at distance L.

- The path difference for two independent rays is

\[ \frac{D}{2} \sin (\theta) \]

- Estimate the location of the first minimum (destructive interference, where \( \lambda/2 \))

\[ \frac{D}{2} \sin (\theta) = \frac{\lambda}{2} \]

- For small \( \theta \) then,

\[ \sin (\theta) = \frac{\lambda}{D} \]

This defines the width of the diffraction pattern and defines the diffraction-limit of 1D aperture.
Diffraction-limit of circular aperture

• The far-field diffraction pattern is given by the Airy function, where $J_1$ is the modified Bessel function of the 1st kind

$$\propto \left[ \frac{J_1(x)}{x} \right]^2,$$

where

$$x \equiv \frac{2\pi D}{\lambda} \sin \theta$$

• The first minimum of the Airy function is at,

$$\sin \theta = 1.22 \frac{\lambda}{D}$$
Point Spread Function

- PSF is the distribution of light intensity in the image of a point source.
- Ideal case: diffraction-limited telescopes – Airy function.
- PSF is a function of the shape of the aperture and obstructions, geometrical optical aberrations, and diffraction effects due to dust and defects on the optics surfaces.

![FWHM Images](image-url)

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II. Basic Optics: raytracing, lenses, & telescopes
Getting focused

• Just as with mirrors, curved lenses follow same rules as flat interfaces, using local surface normal

A lens, with front and back curved surfaces, bends light twice, each diverting incoming ray towards centerline.

Follows laws of refraction at each surface.

Parallel rays, coming, for instance from a specific direction (like a distant star) are focused by a convex (positive) lens to a focal point.

Placing detector at this point would record an image of the star at a very specific spot on the detector. Lenses map incoming angles into positions in the focal plane.
Generating an image

In a pinhole camera, the hole is so small that light hitting any particular point on the image plane must have come from a particular direction outside the camera.

In a camera with a lens, the same applies: that a point on the image plane corresponds to a direction outside the camera. Lenses have the important advantage of collecting more light than the pinhole admits.
Positive Lenses

- Thicker in middle
- Bend rays *toward* axis
- Form *real* focus
Negative Lenses

- Thinner in middle
- Bend rays *away from* the axis
- Form *virtual* focus
Image Formation

- Place arrow (object) on left, trace through image:
  - 1) along optical axis (no deflection – optical axis);
  - 2) parallel to axis (marginal ray), goes through far focus with optical axis ray;
  - 3) through lens center (chief ray);
  - 4) through near-side focus, emerges parallel to optical axis;
  - 5) arbitrary ray with helper
Image Formation

- Note the following:
  - image is inverted
  - image size proportional to the associated s-value:

- Gaussian lens formula (simple form):

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]
Lensmaker’s formula

- Generic lens formula including radius of curvature (R) and index of refraction (n)

\[
\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]
Lenses map directions into displacements

• Two objects at infinity an angle $\Theta$ apart produce distinct spots separated by $\sigma$
Telescope (Refractor w/ eyepiece)

- A telescope has an “objective” lens and an eyepiece
  - sharing a focal plane; giving the eye the parallel light it wants
- Everything goes as ratio of focal lengths: $f_1/f_2$
  - magnification is just $M = f_1/f_2$
  - displacement at focal plane, $\Delta y = \Theta f_2$
  - Notice the light going out is smaller ($P_2$), scales as $P_2 = D_1 f_2/f_1$ – easier to put a eye there or instrument!
Optical aberrations

- Aberrations occur in any optical system due to the wavefront quality of the optics and optical path differences of the rays.
Seidel aberration terms

- We determine the angular deviation of a ray by a third order approximation of sine and cosine.
- The angular aberration (e.g., radians or arcsec) is dependent on radius of curvature (R), height of ray (y), and angle of incidence (θ), and constants (a):

\[
AA_{\text{max}} = \frac{a_s}{f_n^3} + a_c \frac{\theta}{f_n^2} + a_a \frac{\theta^2}{f_n} + a_{fc} \frac{\theta^2}{f_n} + a_d \theta^3
\]

![Diagram of Seidel aberration terms](image)
Distortions at the focal plane

- No distortion
- Pincushion distortion
- Barrel distortion

Distortion of NIRC2 camera on Keck
Newtonian telescope

• The Newtonian has a parabolic mirror and flat secondary mirror that reflects the light to focus at the side of the telescope
  – Used in mostly amateur telescopes today
  – Suffers from coma and astigmatism (one curved surface)
Cassegrain Telescope

- The Cassegrain has a parabolic primary mirror and hyperbolic secondary mirror that focuses the light through a hole in the primary
  - Fixes astigmatism aberrations (and some coma)
  - Folding the optics makes this a compact design
The Hooker telescope on Mt. Wilson – 2.5m Cassegrain telescope

Telescope used by Edwin Hubble to discover the expanding universe
Ritchey–Chrétien Telescope

- An RC telescope has a hyperbolic primary mirror and hyperbolic secondary mirror.
- Many modern telescopes since it fixes coma aberrations.

![Ritchey-Chrétien Telescope](image)

Thirty Meter Telescope w/ M3 tertiary mirror
Focal ratio (f/#, f-ratio)

• The focal ratio characterizes the rate of convergence of a bundle of rays as they form an image

• f/# is the focal Length/ Entrance Pupil Diameter (Input Beam Size)
  - f/# = F_o/D_o (if fully illuminated)

• Intensity in the image plane \( \propto (1/\text{f-ratio})^2 \)
Focal ratio – Optics scale

- High focal ratios (e.g., f/20) imply the incoming rays converge “slow”
- Low focal ratios (e.g., f/2) indicate that the incoming rays converge “fast” at a wide angle
  - Fast systems have increased aberrations and are difficult to focus, but they are incredibly useful for wide-field imaging or broadband spectroscopy
**Aperture Stop**

*Aperture stop*: the element that limits the angular size of the cone of light accepted and so controls the image brightness, e.g., the edge of lenses or an iris diaphragm.
**Field Stop**

*Field stop*: limits the field-of-view (FOV) or the size of the image. Often at focal plane and may be the edge of an imaging sensor.
“Pupils”

• *Pupils* are **images** of physical stops
• *Entrance pupil*: the image of the aperture stop looking forward from object space formed by the intervening lenses
• *Exit pupil*: the image of the aperture stop looking back from image space formed by the intervening lenses.
  – Image resolution and aberrations are often associated with the exit pupil
  – Entrance and exit pupils are **conjugate**, just as an object and image
  – Entrance and exit pupils coincide with the aperture stop at the lens for a single, thin lens—the entrance pupil of an astronomical telescope is typically at the primary mirror.
Consider two “field points” on the focal plane
  – e.g., two stars some angle apart
The rays all overlap at the aperture
  – called the entrance pupil
The rays are separate at the focus (completely distinct)
Then overlap again at exit pupil, behind eyepiece
Optical design constraints

• Often designing an instrument for a specific telescope with a given f/# input
  – \( f/\text{number} = \left( \frac{f_{\text{telescope}}}{D_{\text{telescope}}} \right) \)

• Designing around a specific detector that has particular noise characteristics
  – This means set pixel and array size

• Job is to design optical system with f/# camera to achieve desired plate scale and field of view
Designing a camera to a specific telescope and detector

- If the science cases want a particular field of view and plate scale, then you need to match the re-imaging optics (camera) to the telescope.
Designing an optical seeing-limited camera for Keck (10m)

• Assume that seeing is 0.5” (on Mauna Kea) and we want two pixel sampling, then desired plate scale is 0.25”/pixel

• CCD cameras typically have small pixel sizes, let’s assume $d_{\text{pixel}} = 20 \, \mu m$

$$\frac{f/\#}{\text{camera}} = 206265 \frac{d_{\text{pixel}}}{D_{\text{telescope}} \theta_{\text{pixel}}}$$

$$\frac{f/\#}{\text{camera}} = 206265 \frac{20 \, \mu m}{10 \, m \times 0.25''} = 1.7$$

This is very fast optics!

Optical cameras get more and more challenging to make with bigger telescopes and as detectors use smaller and smaller pixel sizes!

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You are designing two cameras that will each be installed on a 1m and 10m telescope. Each camera uses the same detector and achieve the same plate scale on both telescopes. Which statement is true about the f/# of the cameras?

A. The f/# of the camera for the 1 m telescope is smaller than the f/# for the 10 m telescope.
B. The f/# of the camera for the 1 m telescope is larger than the f/# for the 10 m telescope.
C. The f/#'s of the two cameras are the same.
D. It is not possible to design cameras with the same plate scale and pixel size for a 1 m and a 10 m telescope.
Plate scales

• The size of the detector pixel ($d_{\text{pixel}}$), the diameter of the telescope ($D_{\text{tele}}$), and the (f/#) camera defines the plate scale at the detector

$$\theta_{\text{pixel}} = \frac{206265}{D_{\text{telescope}}(f/\#)_{\text{camera}}} \frac{d_{\text{pixel}}}{D_{\text{tele}}}$$

where $\theta_{\text{pixel}}$ is in units of arcseconds per pixel

• Field of view would then be defined by the detector array size times plate scale
  – E.g., (2048x2048) * $\theta_{\text{pixel}}$
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Good References

From this lecture following input slides from Tom Murphy, Anna Moore, Dae-Sik Moon