The irradiation instability is a disk instability caused by the radiation pressure cast by a central source onto an optically thick disk. The criterion for this instability depends on a sharp transition from an optically thin inner disk to an optically thick outer disk. The quickly diminishing radiation pressure in this transition region creates a radially compressing effect, which is in many ways similar to the gravitational force. In this modal analysis we demonstrate that a disk marginally stable to irradiation can develop global modes, with growth rates being of order the dynamical timescale of the disk. The non-linear evolution of the model shows the formation of vortices near the transition region and spiral structures propagating into the optically thick region. Consequently the scale-height of our disk's inner edge becomes time-variable and can likely be observed as a variation in its infrared flux.

The Criterion for Instability

The condition for a disk to become unstable to irradiation can be derived from a linear analysis using the WKB approximation. The condition for an axisymmetric perturbation to become unstable can be written as:

\[ \frac{d^2\psi}{dr^2} + \left( \frac{\beta}{\kappa} - \frac{r}{\rho} \right) \frac{d\psi}{dr} + \left( \frac{\beta}{\kappa} - \frac{r}{\rho} \right)^2 \psi = 0, \]

where \( \kappa \) is the epicyclic frequency, \( \nu \) is the epicyclic frequency, \( r \) is the optical depth and \( \beta \) is the ratio of radiation pressure to gravity exerted on a mass element. It is important to note, however, that \( \kappa \) is not Keplerian in general as the rotation curve is modified by radiation and gas pressure. Taking into account the modification due to depth and \( \beta \)

\[ 1 - \frac{\beta}{\kappa} \frac{d\ln(\rho)}{dr} - \frac{\ln(\rho)}{r} = 0. \]

This implies if \( \beta \) is constant and less than unity this criterion is never satisfied. However we show in this study that even under this scenario global modes can still develop, similar to marginally stable self-gravitating disks. This work is currently in preparation.

Modal Growth

For \( c_s=0.05, \beta_0=0.1, \) and \( \Delta r=0.05 \) we compute the growth rate of the asymmetric modes by two independent methods: hydrodynamical simulation and semi-analytical solution obtained through solving the linearized equations. The azimuthal number \( m \) is used to decompose the global mode into different linear modes.

On the upper left plot is the result from a numerical simulation. There we measured the amplitude for different \( m^2 \) modes as a function of time, and observe an exponential trend with a growth rate of \(-0.05\) dynamical time \( t_{dyn} \). On the upper right we show the measured growth rates from simulation and calculated growth rates from our semi-analytic solution. With higher resolution (4000 in \( r \) and 600 in \( r \)) the agreement is almost perfect, but even with a lower resolution the agreement is still within 5%.

Non-linear Evolution

At the bottom we show a series of snapshots from our simulation, where the color scale represents the surface density of the disk. At \( t=0 \) orbits, or 565 \( t_{dyn} \), the \( m=4 \) mode is clearly visible. At 100 orbits, the modes are now fully non-linear and 3 vortices are clearly visible along the transition region. These vortices travel at different frequencies, resulting in collisions and agglomeration. At 300 orbits only 2 vortices remain. At the same time it is also noticeable that the sharp transition region has now spread, indicating the irradiation instability can put a limit to the sharpness of an inner disk edge. After 700 orbits only 1 vortex survives. It is likely that the final fate for all global modes in our models is a single vortex, regardless of the initial dominant \( m^2 \) mode. This is because the vortices generated all have the same sign and are all confined to a narrow radial region, making their combination ultimately unavoidable.

We varied three main parameters, \( c_s, \alpha \) and the \( \beta_0 \) to find out how the growth rate of the fastest growing mode scales with them. We find that not only the growth rate changes, the dominant mode also shifts to a different \( m \). In general we observed that the dominant mode has a higher \( m \) if the growth rate is lower.

The plot in the middle right clearly shows the growth rate increases with increasing \( \beta_0 \) as one would expect. Below \( \beta_0=0.06 \) there are no measurable modal growth, confirming our hypothesis that the modal growth is driven by radiation pressure. This plot might imply that the growth rate is a linear function of \( \beta_0 \) but to confirm such a relation would require a more detailed study.

We discover a roughly linear relation between the growth rate and \( c_s/\Delta r \), which is a first order approximation for the second derivative of the gas pressure. This relation is plotted bottom on the right. This suggests the growth rate is most sensitive to the width of the transition region.

References

A Modal Analysis on the Irradiation Instability

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Abstract

We set up a disk model to test the existence of the global modes and compute their growth rates. For simplification this disk has a uniform sound speed \( c_s \), opacity, and \( \beta_0 \). This is justified by the fact that our region of interest, where the disk transitions from optically thin to think, is very narrow, with its width of order the scale height of the disk. The figure below illustrates the surface density with and profile.

We choose a set of standard parameters dictating the disk's properties. We have \( c_s=0.05, \beta_0=0.1, \) and \( \alpha=\text{normalized to } 2/3 \) at \( r=1 \). The units are such that \( GM=1 \) where \( M \) is the mass of the host star. The transition region is created using the error function, and we define the 1-sigma width of the error function to be \( 2\Delta r \), where we choose \( \Delta r=0.05 \). The equation of state is isothermal.

The Setup

The irradiation instability is a disk instability caused by the radiation pressure cast by a central source onto an optically thick disk. This work is currently in preparation.